

MATH SKILLS● **Half-Life**

If 100.0 g of carbon-14 decays until only 25.0 g of carbon is left after 11 460 y, what is the half-life of carbon-14?

1. List the given and unknown values.

Given: initial mass of sample = 100.0 g

final mass of sample = 25.0 g

total time of decay = 11 460 y

Unknown: number of half-lives = ? half-lives

half-life = ? y

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for half-life.

$$\text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of years}}{\text{half - life}}$$

$$\frac{\text{number of years}}{\text{half - life}} = \frac{\text{total time of decay}}{\text{number of half - lives}}$$

3. Calculate how many half-lives have passed during the decay of the 100.0 g sample.

$$\text{fraction of sample remaining} = \frac{\text{final mass of sample}}{\text{initial mass of sample}} = \frac{25.0 \text{ g}}{100.0 \text{ g}} = \frac{1}{4}$$

$$\text{after one half-life} = \frac{1}{2}; \text{ after two half-lives} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ of sample}$$

Two half-lives have passed.

4. Solve for the half-life.

$$\frac{\text{number of years}}{\text{half - life}} = \frac{11\,460 \text{ y}}{2 \text{ half - lives}} = \frac{5730 \text{ y}}{\text{half - life}}$$

$$\text{half-life of carbon-14} = 5730 \text{ y}$$

Your Turn to Think

- What is the half-life of a 100.0 g sample of nitrogen-16 that decays to 12.5 g of nitrogen-16 in 21.6 s?
- All isotopes of technetium are radioactive, but they have widely varying half-lives. If an 800.0 g sample of technetium-99 decays to 100.0 g of technetium-99 in 639 000 y, what is its half-life?
- A 208 g sample of sodium-24 decays to 13.0 g of sodium-24 within 60.0 h. What is the half-life of this radioactive isotope?

MATH SKILLS● **Half-Life** *continued***Sample Problem**

Thallium-208 has a half-life of 3.053 min. How long will it take for 120.0 g to decay to 7.50 g?

1. List the given and unknown values.

Given: half-life = 3.053 min

initial mass of sample = 120.0 g

final mass of sample = 7.50 g

Unknown: number of half-lives = ? half lives

total time of decay = ?

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for the total time of decay.

$$\text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of min}}{\text{half - life}}$$

3. Calculate how many half-lives have passed during the decay of the 120.0 g sample.

$$\text{fraction of sample remaining} = \frac{7.50 \text{ g}}{120.0 \text{ g}} = 0.0625 = \frac{1}{16}$$

$$\text{after one half-life} = \frac{1}{2}; \text{ after two half-lives} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4};$$

$$\text{after three half-lives} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}; \text{ after four half-lives} =$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \text{ of sample. Four half-lives have passed.}$$

4. Solve for the half-life.

$$\text{total time of decay} = 4 \text{ half-lives} \times \frac{3.053 \text{ min}}{\text{half - life}}$$

$$\text{total time of decay} = 12.21 \text{ min}$$

Your Turn to Think

- If the half-life of iodine-131 is 8.10 days, how long will it take a 50.00 g sample to decay to 6.25 g?
- The half-life of hafnium-156 is 0.025 s. How long will it take a 560 g sample to decay to one-fourth its original mass?

MATH SKILLS● **Half-Life** *continued*

6. Chromium-48 has a short half-life of 21.6 h. How long will it take 360.00 g of chromium-48 to decay to 11.25 g

Sample Problem

Gold-198 has a half-life of 2.7 days. How much of a 96 g sample of gold-198 will be left after 8.1 days?

1. List the given and unknown values.

Given: *half-life* = 2.7 days
 total time of decay = 8.1 days
 initial mass of sample = 96 g

Unknown: *number of half-lives* = ? half-lives
 final mass of sample = ? g

2. Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for the number of half-lives.

$$\text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of days}}{\text{half - life}}$$

$$\text{number of half-lives} = \frac{\text{total time of decay}}{\frac{\text{number of days}}{\text{half - life}}}$$

3. Calculate how many half-lives have passed during the decay of the 96 g sample.

$$\text{number of half-lives} = \frac{8.1 \text{ days}}{\frac{2.7 \text{ days}}{\text{half - life}}} = 3.0 \text{ half-lives}$$

4. Calculate how much of the sample will remain after 3.0 half-lives.

$$\text{final mass of sample} = \text{initial mass of sample} \times \text{fraction of sample remaining}$$

$$\text{fraction of sample remaining after three half-lives} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$\text{final mass of sample} = 96 \text{ g} \times \frac{1}{8} = 12 \text{ g}$$

Your Turn to Think

7. Potassium-42 has a half-life of 12.4 hours. How much of an 848 g sample of potassium-42 will be left after 62.0 hours?

MATH SKILLS

● **Half-Life** *continued*

8. Carbon-14 has a half-life of 5730 y. How much of a 144 g sample of carbon-14 will remain after 1.719×10^4 y?
9. If the half-life of uranium-235 is 7.04×10^8 y and 12.5 g of uranium-235 remain after 2.82×10^9 y, how much of the radioactive isotope was in the original sample?