

## Two Dimensional Motion

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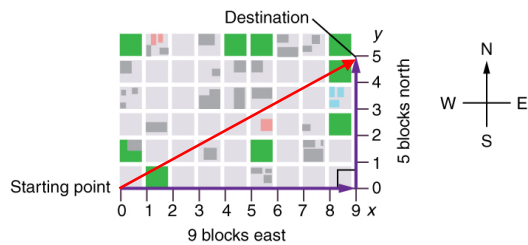
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- A person walks a two-dimensional path between two points.



- The resulting displacement is the straight-line path between the two points.

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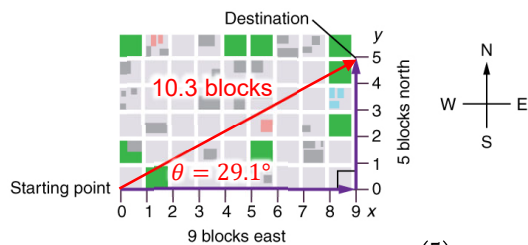
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- The resulting displacement vector can be calculated using the Pythagorean theorem and trigonometry.



$$\sqrt{9^2 + 5^2} = 10.3$$

$$\theta = \tan^{-1}\left(\frac{5}{9}\right) = 29.1^\circ$$

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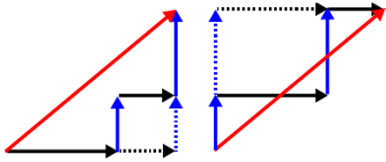
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## Independence of Motion

- The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.



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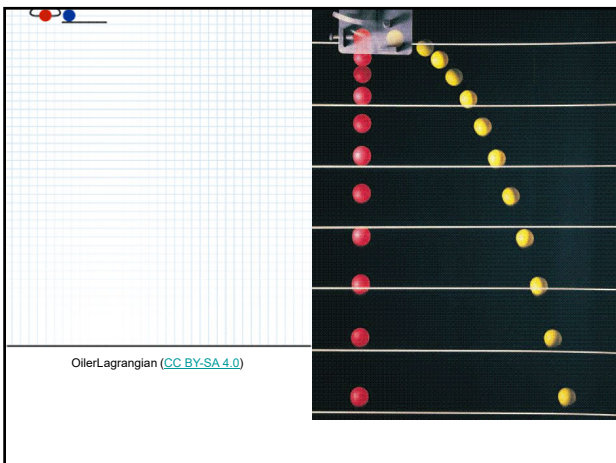
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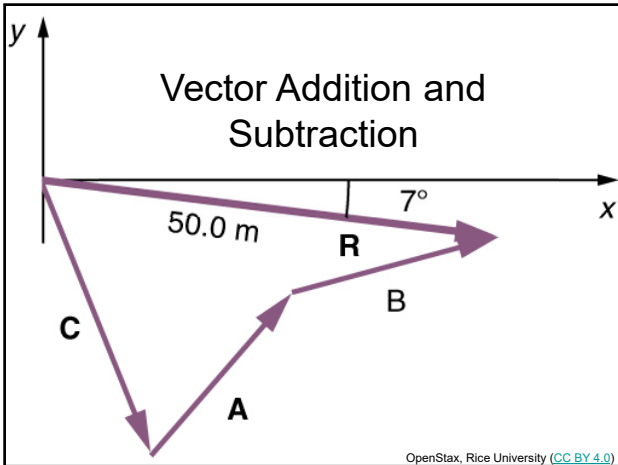
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### Analytical Method

- Vectors will still be represented by arrows of length proportional to the magnitude pointing in the direction of the vector. However, the arrows serve as visual representations only and as such do not have to be to scale.
- Trigonometry and geometry will be used to calculate the resultant vector.

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### Vector Components

- We can take any vector and separate it into its horizontal (x) and vertical (y) components.

The diagram shows a Cartesian coordinate system with x and y axes. A red vector **A** is shown in the first quadrant, making an angle  $\theta$  with the positive x-axis. A right-angled triangle is formed by the vector **A**, its horizontal component **A<sub>x</sub>** along the x-axis, and its vertical component **A<sub>y</sub>** along the y-axis. A dashed red line indicates the vertical component. The equation  $A_x + A_y = A$  is written above the vector. At the bottom right, it says "OpenStax, Rice University (CC BY 4.0)".

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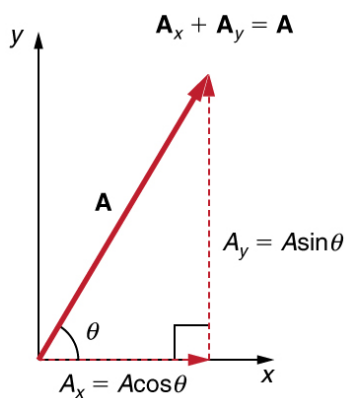
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- If the magnitude (its length) and angle  $\theta$  (its direction) of the vector are known, we can use trigonometry to calculate the components.



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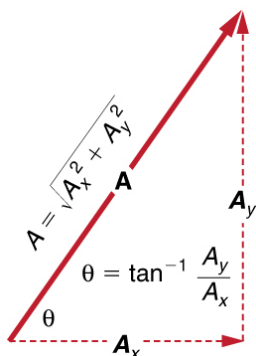
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## Calculating a Resultant Vector

- If the perpendicular components  $A_x$  and  $A_y$  of a vector  $A$  are known, then the magnitude  $A$  and direction  $\theta$  relative to the  $x$ -axis, we can use the Pythagorean theorem and trigonometry.



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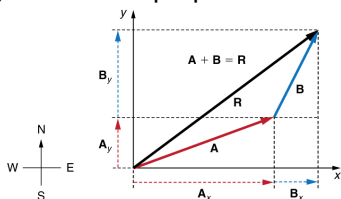
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## Steps

1. Identify the  $x$ - and  $y$ -axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.



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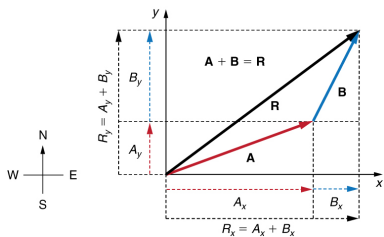
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2. Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.



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3. Calculate the magnitude of the resultant using the Pythagorean theorem.

$$R = \sqrt{R_x^2 + R_y^2}$$

4. Calculate the angle between the x-axis and the hypotenuse using trigonometry.

$$\theta = \tan^{-1} \left( \frac{R_y}{R_x} \right)$$

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### Example

- An airplane has a velocity of 250 m/s  $30^\circ$  north of west. The wind has a velocity of 12 m/s  $25^\circ$  north of east. Calculate the resulting speed of the airplane (known as ground speed).

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	x-component	y-component
Airplane	$-250 \cos 30 = -216.5$	$250 \sin 30 = 125$
Wind	$12 \cos 25 = 10.9$	$12 \sin 25 = 5.1$
Sum	$-205.6$	$130.1$

$v = \sqrt{(205.6)^2 + (130.1)^2}$   
 $v = 243.3 \text{ m/s}$   
 $\theta = \tan^{-1} \frac{130.1}{205.6}$   
 $\theta = 32.3^\circ$

The resultant velocity is 243.3 m/s 32.3° North of West.

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Projectile Motion

LuckySoul (Adobe Stock)

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- When an object travels through the air, the vertical motion can be separated from the horizontal motion.
- Gravity affects the vertical motion of the object causing it to accelerate in the vertical direction.

	Horizontal	Vertical
<b>Acceleration</b>	0	g (9.8 m/s <sup>2</sup> ) down
<b>Velocity</b>	Constant	Changing

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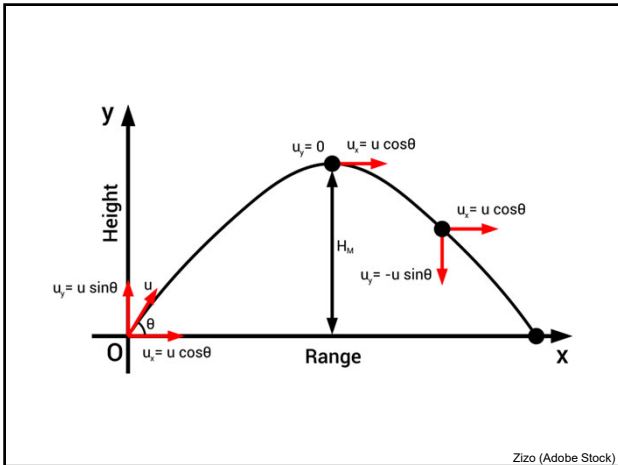
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### Example 1

- A cannon ball is launched with a horizontal velocity of 50 m/s from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

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Separate the horizontal and vertical velocities.

- |                             |                                  |
|-----------------------------|----------------------------------|
| • Horizontal                | • Vertical                       |
| • $v_{xi} = 50 \text{ m/s}$ | • $v_{yi} = 0$                   |
| • $a_x = 0$                 | • $a_y = g = -9.8 \text{ m/s}^2$ |
| • $x = ?$                   | • $y = -10 \text{ m}$            |
| • $t = ?$                   | • $t = ?$                        |

Solve for time,  $t$ , vertically.

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- Vertical

$$y = v_{yi}t + \frac{1}{2}a_yt^2$$

$$t = \sqrt{\frac{2y}{g}}$$

$$t = \sqrt{\frac{2(-10 \text{ m})}{-9.8 \text{ m/s}^2}} = 1.43 \text{ s}$$

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- The time it takes for the object to fall and hit the ground is the same as the horizontal time.
  - The object stops moving horizontally once the object has hit the ground.
- That means that we can now solve for the horizontal distance.

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- Horizontal

$$x = v_{xi}t + \frac{1}{2}a_xt^2$$

$$x = v_{x0}t$$

$$x = (50 \text{ m/s})(1.43 \text{ s}) = 71.5 \text{ m}$$

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## Example 2

- A cannon ball is launched with a velocity of 50 m/s at an angle of  $30^\circ$  from the horizontal from the top of a 10 m high cliff. Calculate the distance from the bottom of the cliff where the cannon ball lands.

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Separate the horizontal and vertical velocities.

- |                            |                                      |
|----------------------------|--------------------------------------|
| • Horizontal               | • Vertical                           |
| • $v_{xi} = 50\cos 30$ m/s | • $v_{yi} = 50\sin 30$ m/s           |
| • $a_x = 0$                | • $a_y = -g = -9.8$ m/s <sup>2</sup> |
| • $x = ?$                  | • $y = -10$ m                        |
| • $t = ?$                  | • $t = ?$                            |

Solve for time,  $t$ , vertically.

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- vertical

$$y = v_{yi}t + \frac{1}{2}a_y t^2$$

$$-10 \text{ m} = (50 \sin 30 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$$

$$4.9t^2 - 25t - 10 = 0$$

$$t = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(4.9)(-10)}}{2(4.9)}$$

$$t = \begin{cases} -0.37 \text{ s} \\ 5.47 \text{ s} \end{cases}$$

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- Horizontal

$$x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t$$

$$x = (50 \cos 30 \text{ m/s})(5.47 \text{ s}) = 237 \text{ m}$$

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## The Monkey and the Hunter



Girl: © MyCuteGraphics.com. Used with permission.  
Monkey: public domain (openclipart.com)

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A hunter with a gun goes out in the woods to hunt for monkeys and sees one hanging in a tree. The monkey releases its grip the instant it hears the gun. Where should the hunter aim to hit the monkey?



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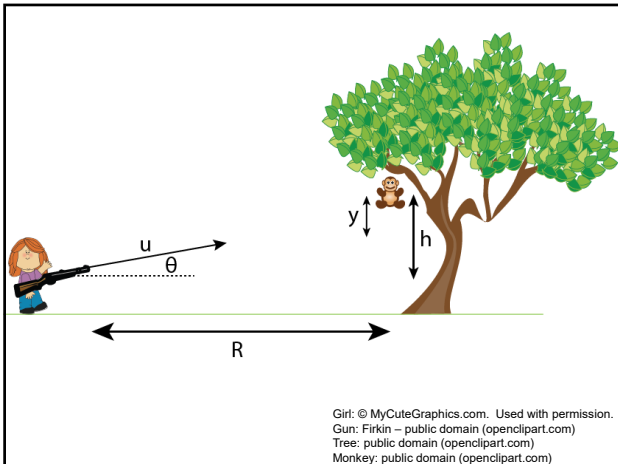
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<p>Mechanics– Projectile Motion Monkey and Hunter</p>	<h1>Monkey and a Gun</h1> <p>MIT Department of Physics Technical Services Group</p>
<p><a href="https://youtu.be/CTSwbyCAGHU?t=368">https://youtu.be/CTSwbyCAGHU?t=368</a></p>	<p><a href="https://youtu.be/cxvsHNRXLjw">https://youtu.be/cxvsHNRXLjw</a></p>

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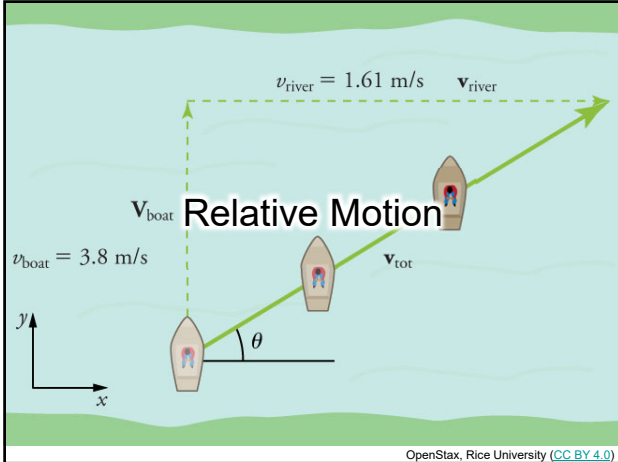
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- If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonally relative to the shore, because the river carries the boat downstream.
- The boat has a velocity relative to a river and the river has a velocity relative to an observer on solid ground.
- The velocity of the boat relative to the observer is the sum of these velocity vectors.

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### Example

A boat with a velocity of 5.0 m/s is crossing a 50.0 m wide river with a current of 2.0 m/s towards the east.

a) What is the velocity of the boat relative to the shore?

b) What is the distance from A to B?

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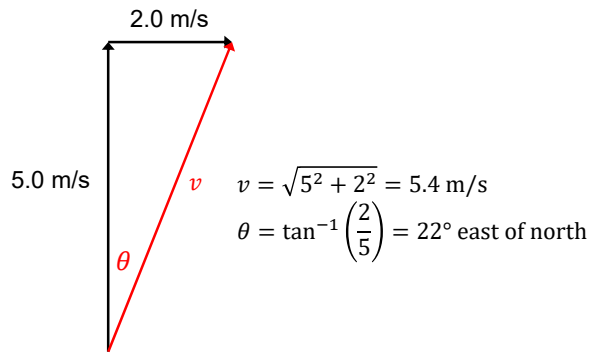
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a) Velocity relative to the shore



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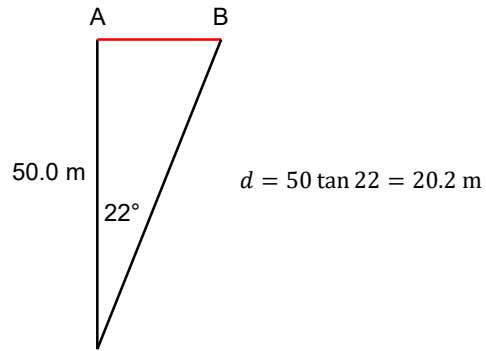
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b) Distance from A to B



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