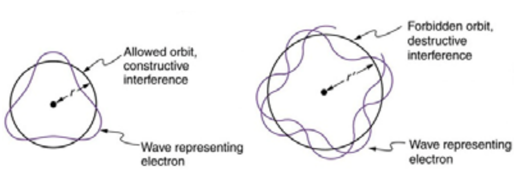
Quantum Mechanics

Electrons as Waves

- de Broglie's hypothesis of electron waves provide an explanation for Bohr's theory of the hydrogen atom
- A particle of mass m moving with speed v would have a wavelength of

$$\lambda = \frac{h}{mv}$$

- Each electron orbit is actually a circular standing wave that closes in on itself
- If the wave does not close in on itself, then destructive interference takes place and the wave quickly dies out



www.boundless.com/physics/textbooks/boundless-physics-textbook/atomic-physics-29/the-early-atom-185/de-broglie-and-the-bohr-model-692-6303/

- The only waves that persist are those for which the circumference of the orbit has a whole number of wavelengths
- So for an orbit of radius r

$$2\pi r = n\lambda$$

$$2\pi r = \frac{nh}{mv}$$

$$mvr = \frac{nh}{2\pi}$$

- The electron is not oscillating in a circular wave, but rather the wave pattern represents the amplitude of the electron wave
- But Bohr's model does not work for atoms other than hydrogen
- A new theory was needed
- That new theory is called quantum mechanics

The Schrödinger Theory

- Erwin Schrödinger (1926) provided a realistic, quantum model for the behavior of electrons in atoms
- There is a wave associated with the electron called the **wavefunction**, $\psi(x,t)$, and is a function of position x and time t

- Given the forces acting on the electron, the wave function can be determined by solving the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t)$$

or the simplified 1 dimensional form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial t^2}$$

- The wavefunction is not directly observable but its amplitude is significant
- The square of the amplitude of the wavefunction is proportional to the probability per unit volume of finding the particle (called probability density)

$$P(r) = |\Psi|^2 \Delta V$$

The Heisenberg Uncertainty Principle

- Werner Heisenberg (1927)
- The principle applied to position and momentum states that it is not possible to measure simultaneously the position ***and*** momentum of something with indefinite precision
- This has nothing to do with imperfect measuring devices or experimental error
- If represents a fundamental property of nature

- The uncertainty Δx in position and the uncertainty Δp in momentum are related by

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

- This says that making momentum as accurate as possible makes position inaccurate, whereas accuracy in position results in inaccuracy in momentum
- If one is made zero, the other has to be infinite

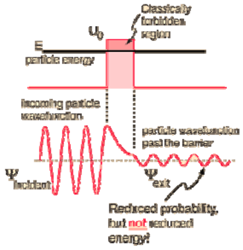
- The uncertainty principle also applies to measurements of energy and time
- If a state is measured to have energy E with uncertainty ΔE, then there must be an uncertainty Δt in the time during which the measurement is made, such that

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

Quantum Tunnelling

- According to classical mechanics, if a particle with energy E approaches a barrier with energy U_0, then the particle will only be able to pass the barrier if $E>U_0$
- However, the wavefunction associated with the particle must be continuous at the barrier and will show an exponential decay through the barrier

- The wavefunction must also be continuous on the other side of the barrier so there is a probability that the particle will tunnel through the barrier



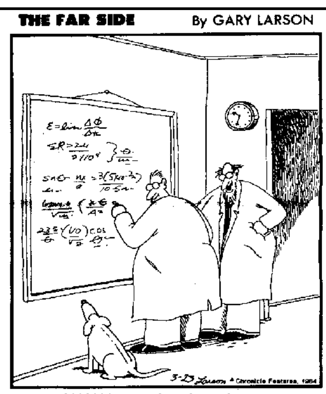
hyperphysics.phy-astr.gsu.edu/hbase/quantum/barr.html

- As a particle approaches the barrier, it is described by a free particle wave function
- When it reaches the barrier, it must satisfy the Schrodinger equation in the form

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} = (E - U_0)\Psi(x)$$

- Which has the solution

$$\Psi = Ae^{-\alpha x} \qquad \text{where} \qquad \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$$



"Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics."